SULIT
3472/2
Matematik Tambahan
Kertas 2
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$2\frac{1}{2}$ jam

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PEPERIKSAAN AKHIR

SIJIL PENDIDIKAN MRSM 2023

PERATURAN PEMARKAHAN

MATEMATIK TAMBAHAN

Kertas 2

Dua jam tiga puluh minit

UNTUK KEGUNAAN PEMERIKSA SAHAJA

AMARAN

Dokumen ini mengandungi 22 halaman bercetak

Additional Mathematics Paper 2
SPMRSM 2023
ANSWER SCHEME

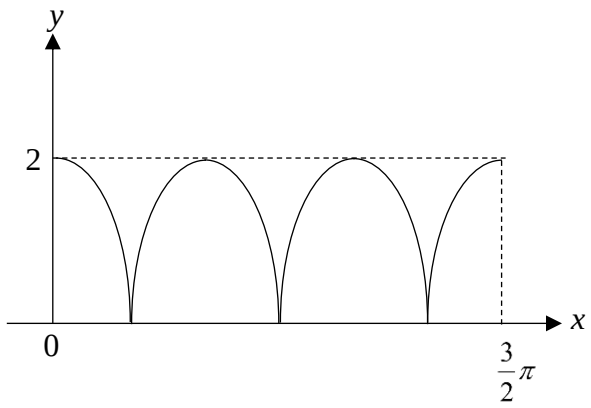
No	Marking Scheme	Sub Marks	Marks
1. (a)	**Method 1:**		
	$3^{4x} = 3^5$		
	$x = \frac{5}{4}$		
	$9^{2\left(\frac{5}{4}\right)-1}$	K1	
	$= 27$	N1	**2**
	Method 2:		
	$9^{2x} = 243$	K1	
	$9^{2x-1} = \frac{9^{2x}}{9}$		
	$= \frac{243}{9}$		
	$= 27$	N1	**2**
	Method 3:		
	$9^{2x} = 9 \times 27$	K1	
	$\frac{9^{2x}}{9} = 27$		
	$9^{2x-1} = 27$	N1	**2**

(b)	(i) $\log_a b = \dfrac{\log_b b}{\log_b a}$	K1	
	$\log_a b = \dfrac{1}{\log_b a}$	N1	**2**
	(ii) $\dfrac{1}{2\log_a b}$ or $\dfrac{3}{2}\log_b a$	K1	
	$\dfrac{3}{2}\log_b a - \dfrac{1}{2}\log_b a$	K1	
	$\log_b a$	N1	**3**

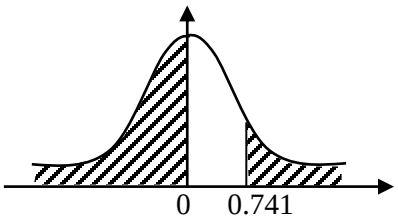
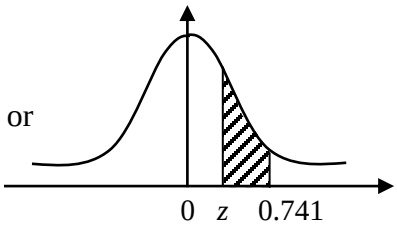
2. (a)	$3g(x)+2=\dfrac{2x+5}{x-2}$ or $f^{-1}[fg(x)]=\dfrac{\left(\dfrac{2x+5}{x-2}\right)-2}{3}$	K1	
	$^{*}\left(\dfrac{3}{(3x+2)-2}\right)$	K1	
	$\dfrac{1}{x}, x\neq 0$	N1	**3**
(b)	$f(x)=0.8x$ or $g(x)=x-100$	P1	
	$fg(x)=0.8(x-100)$	K1	
	$fg(x)=0.8x-80$ and $gf(x)=0.8x-100$	N1	
	Rebate before discount **and** $fg(x)$.	N1	**4**

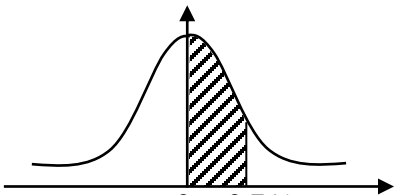
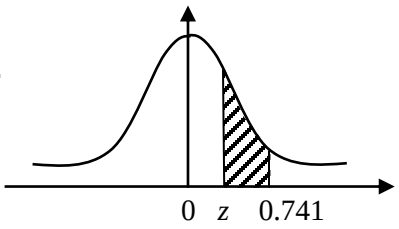
3	$x+y+z=250$ Any one correct equation seen $4x+6y+10z=1550$ $10x+15y+20z=3450$ All equation correct Eliminate first variable using elimination or substitution. Eliminate second variable using elimination or substitution. $z=85$ or $y=20$ or $x=145$ $y=20$ or $x=145$ or $z=85$ x = 145, y = 20, z = 85	P1 P1 K1 K1 N1 N1 N1	**7**
4 (a)	(i) $-2\left[t^2-4t+\left(-\frac{4}{2}\right)^2-\left(-\frac{4}{2}\right)^2-\frac{25}{2}\right]$ OR $t=-\frac{8}{2(-2)}$ and substitute *t into $f(t)$ OR $-2\left[x+\frac{8}{2(-2)}+\frac{4(-2)(25)-8^2}{4(-2)}\right]$ $h(t)=-2(t-2)^2+33$	K1 N1	**2**
	(ii) $t=2$ *Height* = 33	N1 N1	**2**
(b)	$-2(t-2)^2+33=0$ or $-2t^2+8t+25=0$ Equate to 0 and solve. $\therefore t=6.062$	K1 N1	**2**

5 (a)	(i) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$	Write and use triangle law $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ or $\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$ or $\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$ or $\overrightarrow{CE} = \overrightarrow{CD} + \overrightarrow{DE}$	K1	
	$= \underset{\sim}{x} + 4\underset{\sim}{y}$		N1	**2**
	(ii) $\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$			
	$= 5\underset{\sim}{x} - 4\underset{\sim}{y}$		N1	**1**
(b)				
	(i) $\overrightarrow{CE} = h\overrightarrow{CA}$			
	$\overrightarrow{CE} = -h\underset{\sim}{x} - 4h\underset{\sim}{y}$		N1	**1**
	(ii) $\overrightarrow{CE} = \overrightarrow{CD} + k\overrightarrow{DB}$ $= -5\underset{\sim}{x} + k\left(-4\underset{\sim}{y} + 5\underset{\sim}{x}\right)$ $= = (-5 + 5k)\underset{\sim}{x} - 4k\underset{\sim}{y}$		N1	
	$-h = -5 + 5k$ or $-4h = -4k$	Compare the coefficient for $\underset{\sim}{x}$ and $\underset{\sim}{y}$	K1	
		Solve simultaneous linear equations that involve h and k	K1	
	$h = \frac{5}{6}, k = \frac{5}{6}$		N1	**4**

6 (a)	(i) $\sin x\cos 90° + \cos x\sin 90°$ or $\sin x\cos 90° - \cos x\sin 90°$	Use addition formulae	K1	
	$2(\cos x)(-\cos x)+1$ $-\cos 2x$	LHS = RHS	N1	2
	(ii) $\cos 2x = -\frac{1}{2}$	Note: Cannot redo	K1	
	Reference angle $= 60°$ or $2x = 120$		N1	
	$60°, 120°, 240°, 300°$		N1	**3**
(b)		Shape of cosine graph. Accept at least one cycle.	P1	
		Complete 1.5 cycle and $0° \le x \le \frac{3}{2}\pi$	P1	
		Amplitude 2 and absolute	P1	**3**

7 (a)	$-\frac{2x^2}{2}+c$	Integrate $y=-2x$ w.r.t.x	K1	
	$6=-(2)^2+c$	Substitute (2, 6) to find c	K1	
	$y=-x^2+10$ or $g(x)=-x^2+10$		N1	**3**
(b)	$\pi\int_1^6(10-y)dy$			
	$V_1=\pi\left[10y-\frac{y^2}{2}\right]$	Integrate $\pi\int(10-y)\,dy$	K1	
	$*V_1=\pi\left[\left(10(6)-\frac{(6)^2}{2}\right)-\left(10(1)-\frac{(1)^2}{2}\right)\right]$ or $V_2=\frac{1}{3}\pi(2)^2(5)$ or $V_2=\frac{4}{25}\pi\left[\frac{(6-1)^3}{3}-\frac{(1-1)^3}{3}\right]$	Use limit $\int_1^6$ into $*V_1$ or volume of cone or $\pi\int_1^6\frac{4}{25}(y-1)^2\,dy$	K1	
	$^*\frac{65}{2}\pi-{}^*\frac{20}{3}\pi$	Subtract $*V_1-*V_2$ Note: $V_1>V_2$	K1	
	$\frac{155}{6}\pi$ // $25\frac{5}{6}\pi$ // 25.83π		N1	**4**

8 (a)	(i) Continuous random variable	P1	
	(ii) $X \sim N(50, 40.5^2)$ or $X \sim N(50, 1640.25)$	P1	**2**
(b)	**Method 1:**		
	Write $P\left(Z > \frac{80-50}{40.5}\right)$ or $P\left(Z < \frac{50-50}{40.5}\right)$ or $P\left(\frac{q-50}{40.5} < Z < \frac{80-50}{40.5}\right)$	P1	
	$P(Z < 0) + P(Z > 0.741)$ and $0.5 + 0.2293$		
	or	K1	
	Shade and label on the sketching of normal graph  and $0.5 + 0.2293$		
	0.7293	N1	
	0.7293×200 or equivalent	K1	
	145 // 146	N1	**5**
(c)	$P\left(Z > \frac{q-50}{40.5}\right) = 0.15 + 0.2293$ or  and $0.15 + 0.2293$	K1	
	$\frac{q-50}{40.5} = 0.307$ // 0.308	K1	
	62.43 // 62.47	N1	**3**

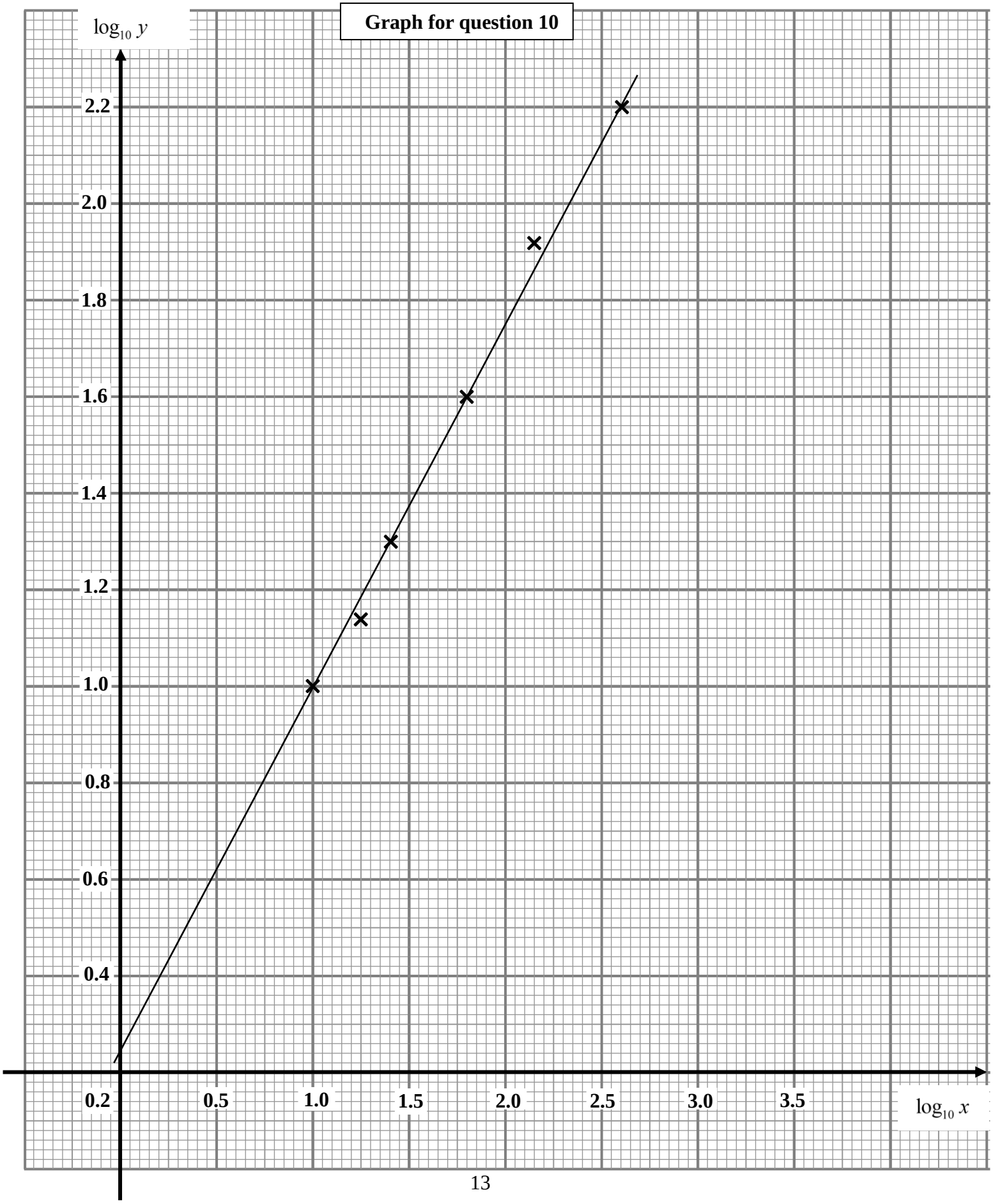
(b)	**Method 2:**		
	Write $P\left(\frac{50-50}{40.5} \le Z \le \frac{80-50}{40.5}\right)$ or $P\left(\frac{q-50}{40.5} < Z < \frac{80-50}{40.5}\right)$	P1	
	$P(0 \le Z \le 0.741)$ and $0.5 - 0.2293$ or Shade and label on the sketching of normal graph  and $0.5 - 0.2293$	K1	
	0.7293	N1	
	0.7293×200 or equivalent	K1	
	145 // 146	N1	**5**
(c)	$P\left(Z > \frac{q-50}{40.5}\right) = 0.15 + 0.2293$ or  and $0.15 + 0.2293$	K1	
	$\frac{q-50}{40.5} = 0.307$ // 0.308	K1	
	62.43 // 62.47	N1	**3**

9				
(a)	$\frac{3(x)+7(1)}{1+3}=1$ or $\frac{3(y)+10(1)}{1+3}=4$	Use divisor of line segment formula. (*cannot be simplified)	K1	
	$A(-1,2)$		N1	**2**
(b)	$\frac{2}{3}x+\frac{26}{3}=6$	Equate to 6 and solve for x.	K1	
	$m_{OP}\times m_{PQ}=\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right)$	Use $m_1 \times m_2 = -1$	K1	
	$=-1$		N1	**3**
(c)	$\frac{2}{3}v+\frac{26}{3}=w$ or equivalent		P1	
	$\frac{1}{2}\left\|\begin{matrix}\left[(-4\times w)+(v\times 2)+(10\times 0)+(0\times 6)\right] \\ -\left[(6\times v)+(10\times w)+(2\times 0)+(0\times -4)\right]\end{matrix}\right\|=114$	Use $\frac{1}{2}\left\|\left(\ \right)-\left(\ \right)\right\|$ and equate 114	K1	
		Solve simultaneous equation to find v or w	K1	
	$w=14$ *or* $v=8$		N1	
	$v=8$ or $w=14$		N1	**5**

10 (a)								
	$\log_{10} x$	1.00	1.25	1.40	1.80	2.15	2.60	N1
	$\log_{10} y$	1.00	1.14	1.30	1.60	1.92	2.20	N1 **2**
(b)	Draw straight line graph $\log_{10} y$ against $\log_{10} x$							
	- Correct axes and uniform scale						Use data given ONLY.	K1
	- At least ONE correct* point plotted							
							6 *points plotted correctly	N1
	Draw line of best fit at least							
	- 5 *points plotted							N1 **3**
(c)	$\log_{10} y = (2k+1)\log_{10} x + \log_{10} h$							P1
	(i) Use $*c = \log_{10} h$							K1
	$h = 1.70$							N1
	Note:							
	$0.21 \le c \le 0.26$							
	(ii) Use $*m = 2k+1$							K1
	$k = -0.125$							N1 **5**

Note:
1. If table is not shown, mark 10(b) and 10(c) according to the correct table.
2. If the axes are interchanged, K0 (M3).
3. If N0 (M4) or N0(M5), then N0N0 (M8,10).
4. SS−1 if not using the given scales.

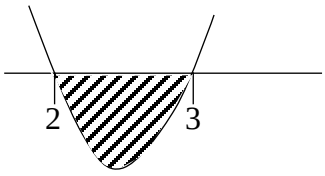
Graph for question 10



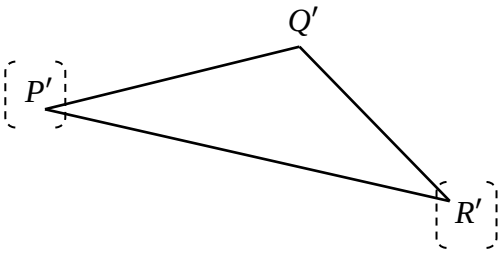
11 (a)	(i) $h = \frac{96-2x^2}{4x}$	Seen or implied	P1	
	$V = x^2 \left(\frac{96-2x^2}{4x}\right) + \frac{1}{3}x^2 \left(\frac{96-2x^2}{4x}\right)$	Substitute * $h = \frac{96-2x^2}{4x}$ into V	K1	
	$V = 32x - \frac{2}{3}x^3$	Note: Must show the simplifying process	N1	**3**
	(ii) $32 - 2x^2 = 0$	Differentiate V w.r.t. x, equate to 0 and solve.	K1	
	$x = 4$		N1	
	$32*(4) - \frac{2}{3}*(4)^3$	Substitute *x into V	K1	
	$\frac{256}{3}$		N1	**4**
(b)	$0.84 = \left[32 - 2(3)^2\right] \times \delta x$		K1	
	$\frac{*0.06}{3} \times 100$		K1	
	2% Note: Do not accept without %		N1	**3**

12 (a)	$\frac{x}{13}\times 100 = 120$ or $\frac{8.75}{7.00}\times 100$	K1	
	$x = 15.60$	N1	**3**
	$y = 125$	N1	
(b)			
	(i) $\frac{120(2)+145(p)+140(2p)+*125(3)}{2+p+2p+3} = 135$ Use $\frac{\sum Iw}{\sum w}$	K1	
	Equate $\frac{\sum Iw}{\sum w}$ to 135 and solve	K1	
	$p = 3$	N1	**3**
	(ii)		
	Method 1:		
	$\frac{100}{125}\times 162$ or $\frac{135\times 125}{100}$	K1	
	$Q_{22} = \text{RM}129.60$		
	$\bar{I}_{\frac{21}{21}}^{\frac{23}{21}} = 168.75$		
	$\frac{100}{135}\times 129.60$ or $\frac{100}{168.75}\times 162$	K1	
	96	N1	
	RM80	N1	**4**

	Method 2: $\frac{\frac{125}{100}\times 135}{168.75}$ $\frac{162\times 100}{120}$ 135 $\frac{*135\times 100}{*168.75}$ 80	K1 K1 K1 NI	

13 (a)	$2t - m$	Differentiate v to find a	K1	
	$2(4) - m = 3$	Substitute $t = 4$ into $*a$, equate 3 and solve.	K1	
	5		N1	**3**
(b)	$t^2 - *5t + 6 < 0$			
	$(t-2)(t+3) < 0$	Use $*v < 0$ and solve	K1	
	and 			
	$2 < t < 3$		N1	**2**
(c)	(i) $\left\lvert \frac{5^3}{3} - 5^2 - 8(5) \right\rvert + 10$ or $\frac{5^3}{3} - 5^2 - 8(5) - 10$		K1	
	$33\frac{1}{3}$		N1	**2**
	(ii) $S_A = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$	Integrate V_A to find S_A	K1	
	$^*\left(\frac{t^3}{3} - \frac{5t^2}{2} + 6t\right) = \frac{t^3}{3} - t^2 - 8t - 10$			
	$3t^2 - 28t - 20 = 0$ $(t-10)(3t+2) = 0$	Equate $S_A = S_B$ and solve *quadratic equation.	K1	
	10		N1	**3**

14 (a)	(i) $\frac{1}{2}\times 16\times 7.3\times \sin\angle PRQ = 32.5$	Use $\frac{1}{2}ab\sin c$	K1	
	146.19°		N1	**2**
	(ii) $7.3^2 + 16^2 - 2(7.3)(16)\cos *146.19°$	Use $a^2 = b^2 + c^2 - 2bc\cos A$	K1	
	22.44		N1	**2**
	(iii) $\frac{\sin\angle RPQ}{7.3} = \frac{\sin *146.19°}{*22.44}$ or $7.3^2 = 16^2 + *22.44^2 - 2(16)(*22.44)\cos P$ or $\frac{1}{2}\times *22.44\times 16\times \sin P = 32.5$	Use $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $a^2 = b^2 + c^2 - 2bc\cos A$ or $\frac{1}{2}ab\sin C = 32.5$	K1	
	10.43° // 10.41°		N1	**2**

(b)	(i)  	NMA Note: 1. $\angle P'Q'R'$ is an obtuse angle 2. Use ruler	N1	**1**
	(ii) 12.95° // 12.97°	NMA	N1	**1**
	(iii) $\frac{1}{2}\times 7.3\times 16\times \sin *12.95°$	Use $\frac{1}{2}ab\sin R'$ or equivalent	K1	
	13.09 // 13.11		N1	**2**

15 (a)	$x + y \leq 70$		N1	
	$y \leq 4x$		N1	
	$y - x \geq 10$		N1	3
(b)	Graph (Attachment)			
	Draw correctly at least one straight line from *inequalities x and y		K1	
	Draw correctly all the *straight line from *inequalities x and/or y Note: accept dotted line and solid line		N1	
	Correct shaded region.	Note: Accept if region labeled **R** No multiple answer	N1	3
(c)	(i) $35 \leq y \leq 45$	Integer only	N1	1
	(ii) (30, 40)		N1	
	$80(*30) + 40(*40)$	Substitute any integer point in the *shaded region into $80x+40y$	K1	
	RM4 000		N1	3
	Note: SS−1 once if (i) In **(a)** more than 3 inequalities given @ x and y not use at all (ii) In **(b)** does not use given scale @ axis interchange			

Graph for question 15