

SULIT
3472/1
Matematik
Tambahan
Kertas 1
November
2023

2 jam



MAKTAB RENDAH SAINS MARA

PEPERIKSAAN AKHIR

SIJIL PENDIDIKAN MRSM 2023

PERATURAN PEMARKAHAN

MATEMATIK TAMBAHAN

Kertas 1

Dua jam

UNTUK KEGUNAAN PEMERIKSA SAHAJA

AMARAN

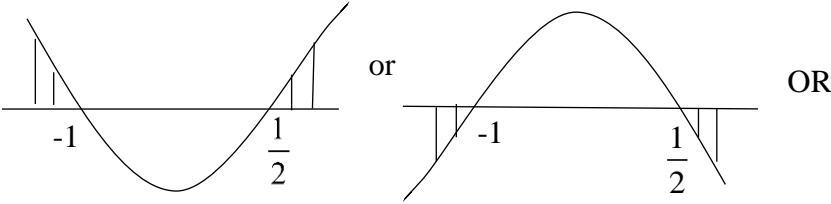
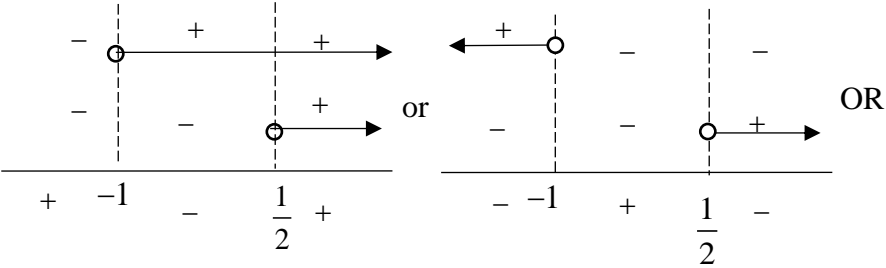
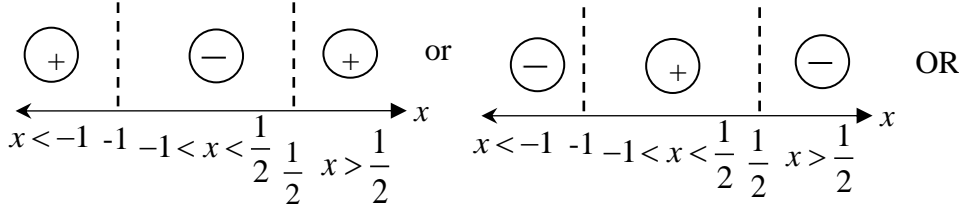
Peraturan pemarkahan ini **SULIT** dan Hak Cipta Bahagian Pendidikan Menengah MARA. Kegunaannya khusus untuk pemeriksa yang berkenaan sahaja. Sebarang maklumat dalam peraturan pemarkahan ini tidak boleh dimaklumkan kepada sesiapa.

Dokumen ini mengandungi 14 halaman bercetak

ANSWER SCHEME

No.	Marking Scheme	Sub marks	Marks
1	$\frac{5 - (-3)}{0 - 4} \text{ OR } -3 = m(4) + 5$ $\frac{y}{x} = (-2)x^2 + 5$ $y = -2x^3 + 5x$	K1 K1 N1	3
2	$\frac{1}{2}(12.5)^2 (2(3.142) - 5.02)$ 98.75 // 98.69	K1 N1	2
3(a)	Differentiate $y = -2x^2 + 10x + 12$ and substitute $x = 4$ <hr style="width: 30%; margin-left: 0;"/> $-4(4) + 10$ -6	K1 N1	
(b)	Use $m_1 \times m_2 = -1$ <hr style="width: 30%; margin-left: 0;"/> $-6 \times m_2 = -1$ Use $y - y_1 = m(x - x_1)$ $\left[y - 20 = \left(\frac{1}{6} \right) (x - 4) \right]$ OR	K1 K1	
	Use $y = mx + c$ and solve for c $\left[20 = \left(\frac{1}{6} \right) (4) + c, c = \frac{58}{3} \right]$ $y = \frac{1}{6}x + \frac{58}{3} \text{ or equivalent}$	N1	5

No.	Marking Scheme	Sub marks	Marks
4(a)	$\left[\frac{3(2^n - 1)}{2 - 1} \right] > 12\,000$ 12	K1 N1	
(b)	<p>ALTERNATIVE A</p> $S_{*12-1} \quad \text{Seen or implied}$ $\left(\frac{3(2^{*11} - 1)}{2 - 1} \right)$ 6141	P1 K1 N1	5
	<p>ALTERNATIVE B</p> <p>Use $T_{*12} = 3(2)^{*12-1}$ AND $S_{*12} = \frac{3(2^{*12} - 1)}{2 - 1}$</p> $S_{*12} - T_{*12}$ $\frac{3(2^{*12} - 1)}{2 - 1} - 3(2)^{*12-1}$ 6141	K1 K1 N1	5

No.	Marking Scheme	Sub marks	Marks																																								
5(a)	<p> $2x^2 + x - 1 > 0$ or $-2x^2 - x + 1 < 0$ or equivalent </p>  <p> $(2x-1)(x+1) > 0$ $(1-2x)(x+1) < 0$ </p>  <p> $(2x-1)(x+1) > 0$ $-(2x-1)(x+1) < 0$ </p>  <p> $x < -1$ -1 $-1 < x < \frac{1}{2}$ $\frac{1}{2}$ $x > \frac{1}{2}$ </p> <table border="1" data-bbox="240 1279 619 1518"> <tr> <td></td> <td></td> <td style="text-align: center;">$\frac{1}{2}$</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">-1</td> <td style="text-align: center;">$\frac{1}{2}$</td> <td></td> </tr> <tr> <td>$(2x-1)$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$(x+1)$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$(2x-1)(x+1)$</td> <td style="text-align: center;">+</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> </table> <p>or</p> <table border="1" data-bbox="703 1279 1098 1518"> <tr> <td></td> <td></td> <td style="text-align: center;">$\frac{1}{2}$</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">-1</td> <td style="text-align: center;">$\frac{1}{2}$</td> <td></td> </tr> <tr> <td>$(1-2x)$</td> <td style="text-align: center;">+</td> <td style="text-align: center;">+</td> <td style="text-align: center;">-</td> </tr> <tr> <td>$(x+1)$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> <td style="text-align: center;">+</td> </tr> <tr> <td>$(1-2x)(x+1)$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> <td style="text-align: center;">-</td> </tr> </table> <p style="text-align: center;">$x < -1, x > \frac{1}{2}$</p> <p><u>Notes:</u></p> <p>For number line and table, must include factorization.</p> <p>For graph, no factorization is needed.</p>			$\frac{1}{2}$			-1	$\frac{1}{2}$		$(2x-1)$	-	-	+	$(x+1)$	-	+	+	$(2x-1)(x+1)$	+	-	+			$\frac{1}{2}$			-1	$\frac{1}{2}$		$(1-2x)$	+	+	-	$(x+1)$	-	+	+	$(1-2x)(x+1)$	-	+	-	<p>P1</p> <p>K1</p> <p>N1</p>	
		$\frac{1}{2}$																																									
	-1	$\frac{1}{2}$																																									
$(2x-1)$	-	-	+																																								
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$(x+1)$	-	+	+																																								
$(1-2x)(x+1)$	-	+	-																																								

No.	Marking Scheme	Sub marks	Marks
(b)	$\left[a \left[x^2 + \frac{b}{a}x + \left(\frac{\left(\frac{b}{a} \right)^2}{2} - \left(\frac{\left(\frac{b}{a} \right)^2}{2} \right) + \frac{c}{a} \right] = 0 \right.$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p style="text-align: center;"><u>Notes :</u></p> $\left(x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2} \text{ must be seen}$	K1 N1	5
6(a)	<p>method to find angle AOB</p> $\cos^{-1} \frac{4}{7} \text{ OR Sine Rule OR } \sin \theta = \frac{\sqrt{33}}{7} \text{ OR Cosine Rule or any valid method}$ <p>0.963</p> <p>(b) 0.963r</p> $2r \sin \left(\frac{55.15}{2} \right) \text{ OR}$ $\sqrt{r^2 + r^2 - 2r^2 \cos 55.15^\circ} \text{ OR}$ $\sqrt{7^2 - 4^2} \text{ or } 7 \sin 55.15^\circ \text{ or } 4 \tan 55.15^\circ \text{ OR Sine Rule OR Cosine Rule}$ <p>or any valid method to find AB or chord PR</p> $0.963r + 2r \sin \frac{55.15}{2} < 7 + 4 + \sqrt{7^2 - 4^2}$ <p>$7 < r < 8.865$</p>	K1 N1 K1 K1 K1 N1	6

<p>7(a)</p> <p>(b)</p> <p>(c)</p>	<p>6</p> <p>$\vec{OQ} = \vec{OP} + \vec{PQ}$ OR</p> <p>$\vec{OQ} = \vec{OR} + \vec{RQ}$ OR</p> <p>$\vec{RN} = \vec{RQ} + \vec{QN}$ OR</p> <p>$\vec{RN} = \vec{RO} + \vec{ON}$</p> <p>$\tilde{p} + \tilde{r}$</p> <p>$\frac{1}{2}\tilde{p} - \frac{1}{2}\tilde{r}$</p> <p>$\sqrt{2^2 - 1^2}$ or $2\cos 30^\circ$ or $2\sin 60^\circ$</p> <p>$3.464 // 2\sqrt{3}$</p>	<p>P1</p> <p>K1</p> <p>N1</p> <p>N1</p> <p>K1</p> <p>N1</p>	<p>6</p>
<p>8</p> <p>(a)(i)</p> <p>(ii)</p>	<p>24</p> <p>${}^2C_1 \times {}^7C_2$ or ${}^2C_2 \times {}^7C_1$ OR</p> <p>9C_3 or 7C_3</p> <p>${}^2C_1 \times {}^7C_2 + {}^2C_2 \times {}^7C_1$ OR</p> <p>${}^9C_3 - {}^7C_3$</p> <p>49</p>	<p>N1</p> <p>K1</p> <p>K1</p> <p>N1</p>	

<p>(b)(i)</p> <p>(ii)</p>	<p>1320</p> <p>${}^4P_1 \times 3!$ or ${}^3P_2 \times {}^9P_1 \times 8$ or ${}^4C_1 \times {}^3C_1 \times (2 \times {}^9P_1)$ or ${}^4P_3 \times {}^3P_1 \times {}^3P_1 \times {}^3P_1$</p> <p>1320 - ${}^4P_1 \times 3!$ - ${}^3P_2 \times {}^9P_1 \times 8$ OR ${}^4C_1 \times {}^3C_1 \times (2 \times {}^9P_1) + {}^4P_3 \times {}^3P_1 \times {}^3P_1 \times {}^3P_1$</p> <p>864</p>	<p>N1</p> <p>K1</p> <p>K1</p> <p>N1</p>	<p>8</p>
<p>9(a)</p> <p>(b)</p>	<p>$y = -x^3 - x^2 + 2x$</p> <p>$-x^3 - x^2 + 2x = 0$</p> <p>$x = 0, -2, 1$</p> <p>Integrate * y wrt x and use limit \int_{*-2}^0 or \int_0^{*1}</p> <p>$I_1 + I_2 ; I_1 < 0$</p> <hr/> <p>$\left -\frac{8}{3} \right + \frac{5}{12}$</p> <p>$\frac{37}{12}$</p>	<p>N1</p> <p>K1</p> <p>K1</p> <p>K1</p> <p>N1</p>	<p>5</p>

No.	Marking Scheme	Sub marks	Marks
10(a)	3:1	P1	
	$\left(\frac{1(-4)+3(8)}{3+1}, \frac{1(4)+3(-12)}{3+1} \right)$	K1	
	(5, -8)	N1	
	(b) Find gradient PQ or $*SP$ or $*SQ$ and use $m_1 \times m_2 = -1$	K1	
	Use $y - y_1 = m(x - x_1)$	K1	
	$\left[y - (-8) = \left(\frac{3}{4} \right) (x - 5) \right]$ <p style="text-align: center;">OR</p> <p>Use $y = mx + c$ and solve for c</p> $\left[-8 = \left(\frac{3}{4} \right) (5) + c \right], c = -\frac{47}{4}$		
$y = \frac{3}{4}x - \frac{47}{4} \text{ or equivalent}$	N1	6	

No.	Marking Scheme	Sub marks	Marks
11(a)	<p>Arithmetic progression is a sequence of numbers such that each term is obtained by adding a constant to the term before it.</p> <p>Janjang aritmetik ialah suatu jujukan nombor dengan setiap sebutan diperoleh dengan menambahkan satu pemalar kepada sebutan sebelumnya.</p>	P1	
(b)(i)	$\frac{15}{2}[2a + (15-1)d] = 360 \text{ or } \frac{15}{2}(a + 38) = 360$ <p style="text-align: center;">and</p> $a + (15-1)d = 38$ <p>M₁ $a = 10$ and $d = 2$ M₂ $d = 2$ M₃ $d = 2$</p> <p>M₁ $360 - \left[\frac{10}{2} (2)(10) + (10-1)(2) \right]$ OR M₂ $\frac{5}{2} [2(38) + (5-1)(-2)]$ or $\frac{5}{2} [2(30) + (5-1)(2)]$ OR M₃ $[30 + 32 + 34 + 36 + 38]$</p> <p>170</p>	K1 N1	
(ii)	$\frac{15}{2}[2a + 14d] = 360$ <p>$d = 1$ & $a = 17$ $d = 3$ & $a = 3$</p>	K1 N1 N1	8

No.	Marking Scheme	Sub marks	Marks
12(a)	Use ${}^3C_3 p^3 (1-p)^0 = 512s$ or ${}^3C_0 p^0 (1-p)^3 = 8s$	K1	
	$8\sqrt[3]{s}$	N1	
	(b) $8\sqrt[3]{s} = 1 - 2\sqrt[3]{s}$ and solve for s	K1	
	${}^3C_2 (8\sqrt[3]{0.001})^2 (1 - 8\sqrt[3]{0.001})$ OR ${}^3C_2 (1 - 2\sqrt[3]{0.001})^2 (1 - (1 - 2\sqrt[3]{0.001}))$ or equivalent 0.384	K1 N1	5

No.	Marking Scheme	Sub marks	Marks
13			
(a)(i)	17	N1	
(ii)	3	N1	
(iii)	$\frac{x+4}{3} = x$	K1	
	$x = 2$	N1	
(b)	$g^{-1}(x) = 3x - 4$ & $f(x) = \frac{18}{17 - (3x - 4)}$ OR	K1	
	$f(y) = \frac{18}{17 - (3y - 4)}$		
	$\frac{6}{7-x}, \{x \neq 7\}$	N1	
(c)	$\left(\frac{6}{7 - \left(7 - \frac{6}{x} \right)} \right)$ OR $7 - \frac{6}{\left(\frac{6}{7-x} \right)}$	K1	
	$fh(x) = hf(x) = x$	N1	8

No.	Marking Scheme	Sub marks	Marks
14(a)	$\pi \left(\frac{1}{\sqrt{3}+1} \right)^2 (\sqrt{3}-1)$ $\pi \left(\frac{\sqrt{3}-1}{4+2\sqrt{3}} \right) \times \left(\frac{4-2\sqrt{3}}{4-2\sqrt{3}} \right)^*$ $\pi \left(\frac{4\sqrt{3}-2(3)-4+2\sqrt{3}}{16-12} \right)^*$ $\left(-\frac{5}{2} + \frac{3}{2}\sqrt{3} \right) \pi$ <p><u>Notes:</u></p> <ol style="list-style-type: none"> The numerator and denominator must contain at least 2 terms with a surd If there is only 1 term in numerator, K0N0 	<p>P1</p> <p>K1</p> <p>K1</p> <p>N1</p>	
(b)	$r = \frac{1}{2} \left(\ln \frac{33900000}{33200000} \right)$ <p>1.043</p>	<p>K1</p> <p>N1</p>	
(c)	<p>Use $(a^m)^n = a^{mn}$</p> <p>$(5^p)^r$ or $(3^q)^r$</p> <p><u>Notes:</u></p> <p>$p \times r = 300$, $q \times r = 500$</p> <p>where p, q and $r \in \mathbb{R}$</p> <p>Hazwan, $5^p < 3^q$</p>	<p>K1</p> <p>N1</p>	<p>8</p>

<p>15(a)</p>	$21(1 - \sin^2 x) = 8 \sin x + 16$ $(3 \sin x - 1)(7 \sin x + 5) = 0$ <p>19.47°, 45.58° // 19°28', 45°35'</p> <p>19.47°, 160.53°, 225.58°, 314.42° // 19°28', 160°32', 225°35', 314°25'</p> <p>(b) ALTERNATIVE A</p> $\tan 50^\circ = \frac{h}{p}$ $\left(\frac{h}{p} \right) = \frac{2 \tan 25^\circ}{1 - \tan^2 25^\circ}$ $\frac{-2p \pm \sqrt{(2p)^2 - 4(h)(-h)}}{2h} \text{ OR}$ $\tan^2 25^\circ + \frac{2p}{h} \tan 25^\circ + \left(\frac{2p}{h} \right)^2 - \left(\frac{2p}{h} \right)^2 - 1 = 0$ $\frac{-p + \sqrt{p^2 + h^2}}{h}$	<p>K1</p> <p>K1</p> <p>N1</p> <p>N1</p> <p>P1</p> <p>K1</p> <p>K1</p> <p>N1</p>	<p>8</p>
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No.	Marking Scheme	Sub marks	Marks
	<p>ALTERNATIVE B</p> $\sqrt{p^2 + h^2}$ $\cos 50^\circ = \frac{p}{\sqrt{p^2 + h^2}}$ $\frac{\sqrt{\frac{1 - \cos 50^\circ}{2}}}{\sqrt{\frac{\cos 50^\circ + 1}{2}}} \quad \text{or} \quad \frac{\sqrt{1 - \frac{p}{\sqrt{h^2 + p^2}}}}{\sqrt{1 + \frac{p}{\sqrt{h^2 + p^2}}}}$ $\frac{-p + \sqrt{p^2 + h^2}}{h}$ <p>ALTERNATIVE C</p> $\sqrt{p^2 + h^2}$ $\cos 50^\circ = \frac{p}{\sqrt{p^2 + h^2}} \quad \text{or} \quad \sin 50^\circ = \frac{h}{\sqrt{p^2 + h^2}}$ $\frac{\frac{h}{\sqrt{h^2 + p^2}}}{1 + \frac{p}{\sqrt{h^2 + p^2}}}$ $\frac{-p + \sqrt{p^2 + h^2}}{h}$	<p>P1</p> <p>K1</p> <p>K1</p> <p>N1</p> <p>P1</p> <p>K1</p> <p>K1</p> <p>N1</p>	<p>8</p> <p>8</p>