

SULIT

3472/2

Matematik

Tambahan

Kertas 2

November

2022

2 jam 30 min



MAKTAB RENDAH SAINS MARA

PEPERIKSAAN AKHIR

SIJIL PENDIDIKAN MRSM 2022

PERATURAN PEMARKAHAN

MATEMATIK TAMBAHAN

Kertas 2

Dua jam tiga puluh minit

UNTUK KEGUNAAN PEMERIKSA SAHAJA

AMARAN

Peraturan pemarkahan ini **SULIT** dan Hak Cipta Bahagian Pendidikan Menengah MARA. Kegunaannya khusus untuk pemeriksa yang berkenaan sahaja. Sebarang maklumat dalam peraturan pemarkahan ini tidak boleh dimaklumkan kepada sesiapa.

Dokumen ini mengandungi 22 halaman bercetak

Additional Mathematics Paper 2
SPMRSM 2022
ANSWER SCHEME

No	Solution	Scheme	Sub Marks	Marks
1	<p>Method: Substitution $y = -x - 5$ or $x = -y - 5$</p> <p>$2(-x - 5)^2 + x = 5$ or $2y^2 + (-y - 5) = 5$</p> <p>$x = -\frac{15}{2}, x = -3$ @ $y = \frac{5}{2}, y = -2$</p> <p>$y = \frac{5}{2}, y = -2$ and $x = -\frac{15}{2}, x = -3$</p>	<p>P1 Seen or implied</p> <p>K1 Substitute *y or *x into the non-linear equation.</p> <p>K1 Solve the quadratic equation $ax^2 + bx + c = 0$, by using factorization OR quadratic formula OR completing the square.</p> <p>Factorization $(2x + 15)(x + 3) = 0$ or $(2y - 5)(y + 2) = 0$</p> <p>OR</p> <p>Completing the square $2 \left[x^2 + \frac{21}{2}x + \left(\frac{21}{2}\right)^2 - \left(\frac{21}{2}\right)^2 + \frac{45}{2} \right] = 0$ or $2 \left[y^2 + \frac{y}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 5 \right] = 0$</p> <p>OR</p> <p>Quadratic Formula $x = \frac{-21 \pm \sqrt{(21)^2 - 4(2)(45)}}{2(2)}$ or $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-10)}}{2(2)}$</p> <p>a, b, c must be correct</p>		

Method: Elimination

$$4y^2 + 2x = 10 \quad \text{or} \quad y + x = -5$$

$$2y + 2x = -10 \quad \text{or} \quad y + x = -5$$

$$(-) \quad \frac{4y^2 + 2x = 10}{2y - 4y^2 = -20} \quad (-) \quad \frac{2y^2 + x = 5}{y - 2y^2 = -10}$$

$$x = -\frac{15}{2}, x = -3 \quad \text{or} \quad y = \frac{5}{2}, y = -2$$

$$y = \frac{5}{2}, y = -2 \quad \text{and} \quad x = -\frac{15}{2}, x = -3$$

P1

K1

Eliminate one variable.

K1

Solve the quadratic equation $ax^2 + bx + c = 0$, by using factorization **OR** quadratic formula **OR** completing the square.

Factorization

$$(2y - 5)(y + 2) = 0 \quad \text{or} \quad 2(2y - 5)(y + 2) = 0$$

OR

Completing the square

$$\text{or } 2 \left[y^2 + \frac{y}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 5 \right] = 0$$

OR

Quadratic Formula

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-10)}}{2(2)}$$

a, b, c must be correct

N1

N1

5

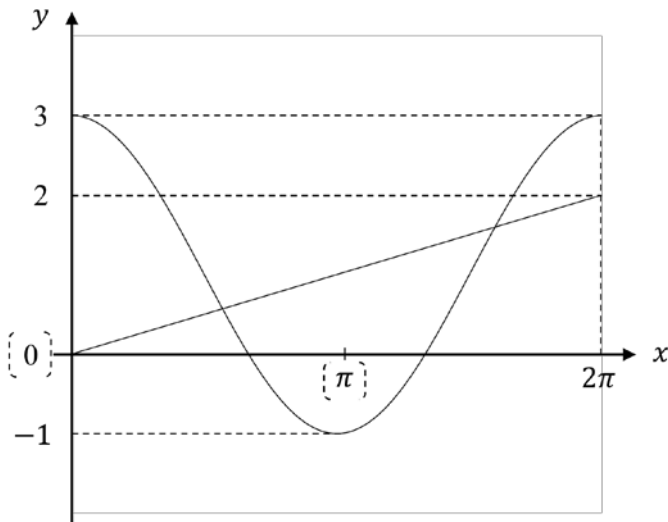
5

2
(a)

$$\operatorname{cosec}^2 x = 1 + \cot^2 x \quad \text{or} \quad \sin^2 x + \cos^2 x = 1$$

$$\cos^2 x$$

(b) (i)



(ii)

$$y = \frac{x}{\pi}$$

Refer question 2 (b) (i)

2

(K1) Use or implied

(N1) LHS = RHS

2

(P1) Shape of cosine graph

(P1) Complete 1 cycle **and** 2π must be seen.

3

(P1) Amplitude 4 **and** shift upward 1 unit

Note:
Maximum value = 3
Minimum value = -1

(N1)

3

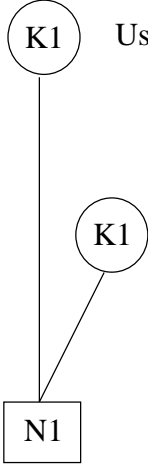
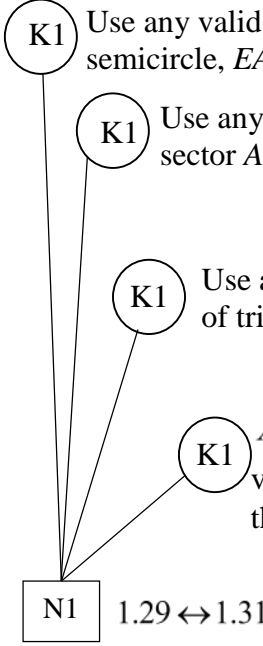
(K1) Sketch straight line $y = \frac{x}{\pi}$
with correct gradient and y-intercept

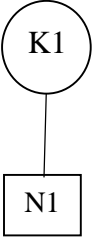
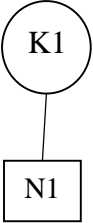

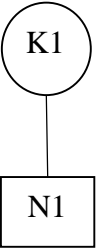

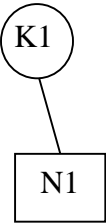
(N1)

<p>3 (a)</p>	$[3x^2 - x]_0^1$ $(3(1)^2 - (1)) - (3(0)^2 - (0))$ <p style="text-align: center;">2</p>	<p>P1</p> <p>K1 Use the limit of \int_0^1 into $3x^2 - x$ Note: Must show completely</p> <p>N1</p>	<p style="text-align: center;">3</p>	
<p>(b)</p>	$3(5) - h = \frac{7.5 - 0}{5 - 3.75}$ $7.5 = \frac{3}{2}(5)^2 - 9(5) + c$ $y = \frac{3}{2}x^2 - 9x + 15$	<p>K1 $3(5) - h = \frac{7.5 - 0}{5 - 3.75}$</p> <p>K1 Integrate $3x - 9$ and substitute to find c.</p> <p>N1</p>	<p style="text-align: center;">3</p>	<p style="text-align: center;">6</p>

<p>4 (a)</p>	<p>$\frac{5}{2} + 3(2^{1-1} - 1)$ or $\frac{5}{2} + 3(2^{2-1} - 1)$</p> <p>$\frac{5}{2} // 2.5$</p> <p>$\frac{\left(\frac{11}{2}\right) - \left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$</p> <p>$\frac{6}{5} // 1.2$</p>	<p>Use S_n Note: Can reward K1 in (a)(ii) if not seen in (a)(i)</p> <p>Use $r = \frac{T_{n+1}}{T_n}$</p>	<p>4</p>	
<p>(b)</p>	<p>$\frac{9}{10}$</p> <p>$\frac{1}{1 - \left(\frac{9}{10}\right)}$</p> <p>10</p>	<p>Use $S_\infty = \frac{a}{1-r}$</p>	<p>3</p>	<p>7</p>

<p>5 (a)</p>	<p>(6,5)</p> $\sqrt{(*6-2)^2 + (*5-7)^2} @ \sqrt{(-2-2)^2 + (9-7)^2}$ <p>@</p> $\sqrt{(x-*6)^2 + (y-*5)^2}$ $\sqrt{(-2-2)^2 + (9-7)^2} = \sqrt{(x-*6)^2 + (y-*5)^2}$ <p>@</p> $\sqrt{(*6-2)^2 + (*5-7)^2} = \sqrt{(x-*6)^2 + (y-*5)^2}$ <p>@</p> $\sqrt{20} = \sqrt{(x-*6)^2 + (y-*5)^2}$ $x^2 + y^2 - 12x - 10y + 41 = 0$	<p>P1 Implied</p> <p>K1 Find distance</p> <p>K1 Equate BS=SQ</p> <p>N1 Note: MUST in lowest term with '= 0'.</p>	<p>4</p>	
<p>(b)</p>	$\frac{2(3) + x(1)}{1+3} = *6 @ \frac{7(3) + y(1)}{1+3} = *5$ <p>(18, -1)</p> $A = \frac{1}{2} [0(3) + 0(7) + 2(*-1) + *18(0)] - [0(0) + 2(3) + *18(7) + 0(*-1)]$ <p>67</p>	<p>K1</p> <p>N1 (18, -1)</p> <p>K1</p> <p>N1 67</p>	<p>4</p>	<p>8</p>

<p>6 (a)</p>	$4.754 = OA(113.56 \times \frac{\pi}{180})$ $OA = 2.398$ $BA = * 5.50 \text{ or } BO = * 6.00$ <p>0.50</p>	 <p>Use $s = r\theta$</p> <p>Use any valid method to find BA or BO.</p>	<p>3</p>	
<p>(b)</p>	<p>4.012</p> $A_{AOC} = \frac{1}{2} * (2.398)^2 \left(66.44 \times \frac{3.142}{180} \right) \text{ or}$ $A_{APD} = \frac{1}{2} * (4.012)^2 \left(33.22 \times \frac{3.142}{180} \right)$ $A_{OAP} = \frac{1}{2} * (2.398) * (2.398) \sin 113.56$ $A_{AOC} + A_{AOP} - A_{APD}$ <p>1.29 ↔ 1.31</p>	 <p>Use any valid method to find radius of semicircle, EAD.</p> <p>Use any valid method to find area of sector AOC or APD.</p> <p>Use any valid method to find area of triangle AOP</p> <p>$A_{AOC} + A_{AOP} - A_{APD}$ or any valid method to find area of the shaded region.</p> <p>1.29 ↔ 1.31</p>	<p>5</p>	<p>8</p>

<p>8</p> <p>(a)</p> <p>(i)</p> <p>$np = 20, npq = 16$ (both) or $20q = 16$</p> <p>$0.2 // \frac{1}{5}$</p> <p>(ii)</p> <p>${}^{10}C_3 (0.2)^3 (0.8)^7 // {}^{10}C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$</p> <p>0.2013</p> <p>(b)</p> <p>(i)</p> <p>$\left\{ - \right\} 0.16$</p> <p>$\frac{72 - \mu}{50} = -0.16$</p> <p>80</p> <p>(ii)</p> <p>0.0999</p> <p>(iii)</p> <p>$349 \times \text{RM}150$ or $350 \times \text{RM}150$</p> <p>52350 // 52500</p>	<p>(i)</p>  <p>(ii)</p>  <p>0.2013</p> <p>(i)</p>  <p>(ii)</p>  <p>Use of $Z = \frac{x - \mu}{\sigma}$</p> <p>(ii)</p>  <p>(iii)</p> 	<p>4</p> <p>6</p>	<p>10</p>
---	---	-------------------	-----------

9

(a)

(i)

$$5y$$

$$\overline{AC} = \overline{AB} + \overline{BC}$$

$$4x + 5y$$

(ii)

$$\frac{12}{5}x$$

$$\left(3y + \frac{12}{5}x\right) = k(4x + 5y)$$

$$3 = 5k \quad \text{or} \quad \frac{12}{5} = 4k$$

$$\frac{3}{5}$$

P1

K1

Write triangle law

N1

K1

K1

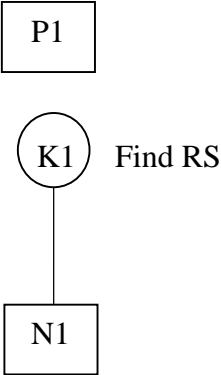
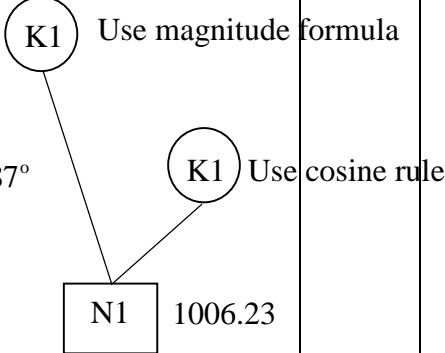
$$\left(3y + \frac{12}{5}x\right) = k(4x + 5y)$$

K1

Compare the coefficient of x or y

N1

7

<p>(b)</p>	<p>1800</p> $RS = \sqrt{1800^2 + 900^2}$ <p>1006.23</p> <p><u>Alternative method</u></p> $PR = \sqrt{900^2 + 1200^2}$ $RS = 3000^2 + (\sqrt{900^2 + 1200^2})^2 - 2(3000)(\sqrt{900^2 + 1200^2}) \cos 36.87^\circ$ <p>1006.23</p>	 <p>P1</p> <p>K1 Find RS</p> <p>N1</p>  <p>K1 Use magnitude formula</p> <p>K1 Use cosine rule</p> <p>N1 1006.23</p>	<p>3</p>	<p>10</p>
------------	---	--	----------	-----------

10
(a)

$\frac{1}{y}$	1.11	2.00	2.94	4.35	5.56	6.25
---------------	------	------	------	------	------	------

N1 At least 2 d.p

Draw straight line graph $\frac{1}{y}$ against x

- Correct axes and uniform scale
- At least ONE correct * point plotted

Draw line of best fit at least

- 5 *points plotted
- Table in 2 d.p

(b) i) $0.36 \leftrightarrow 0.39$

ii) $\frac{1}{y} = \frac{2}{d}x - \frac{f}{d}$

$\frac{2}{d} = *m$

$4.65 \leftrightarrow 4.88$

$-\frac{f}{*d} = *c$

Note:

1. If the table of values is not shown, then check values from the graph.
2. If the axes are interchanged, N1 for table of values only

$3.33 \leftrightarrow 3.54$

K1 Use data given ONLY.

N1 6 *points plotted correctly

N1 4

N1

P1

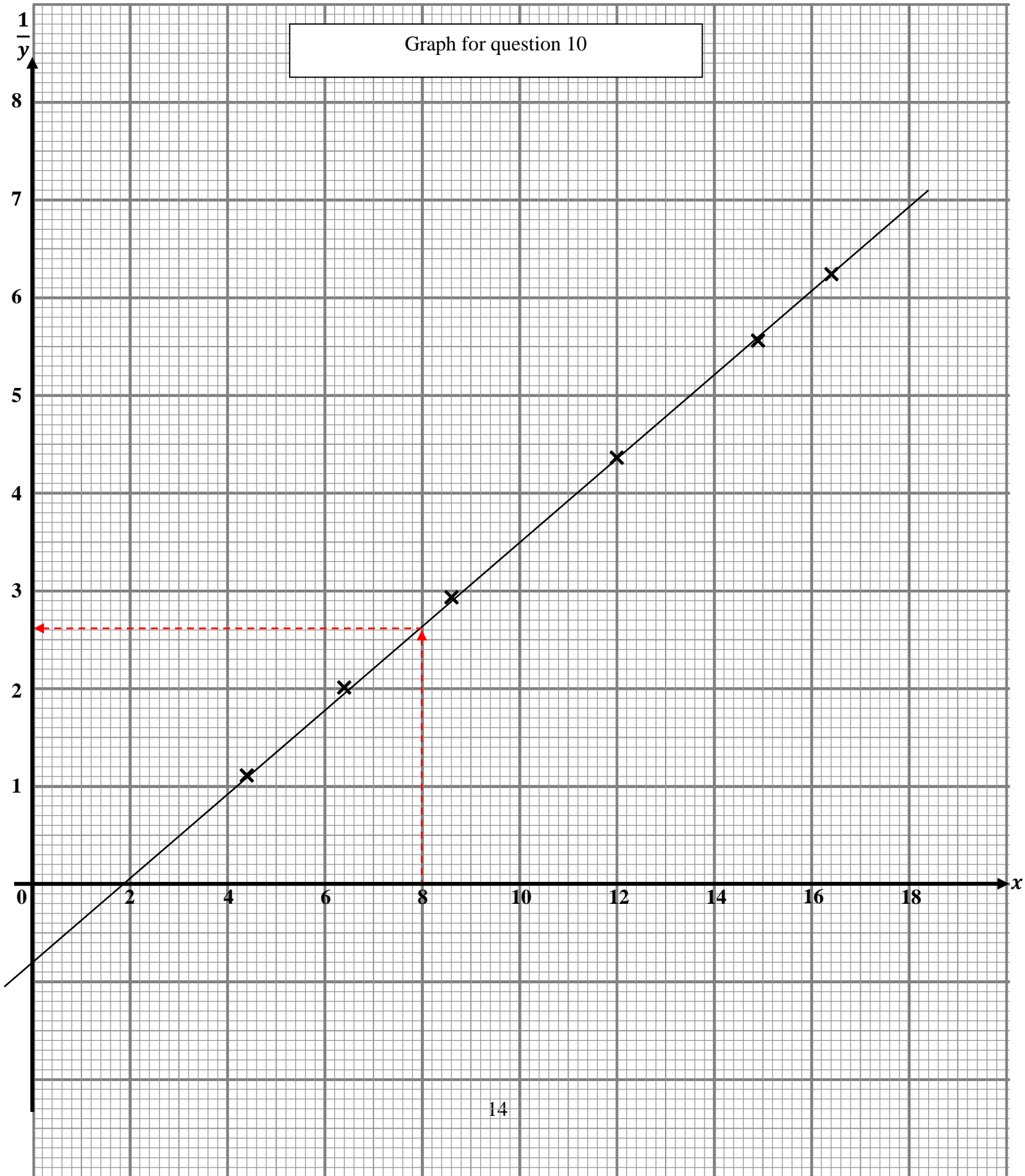
K1
N1 4.65 ↔ 4.88

K1 6

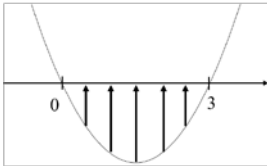
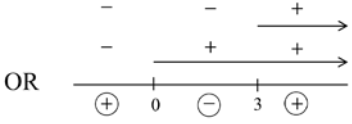
K1
N1 3.33 ↔ 3.54

10

Graph for question 10

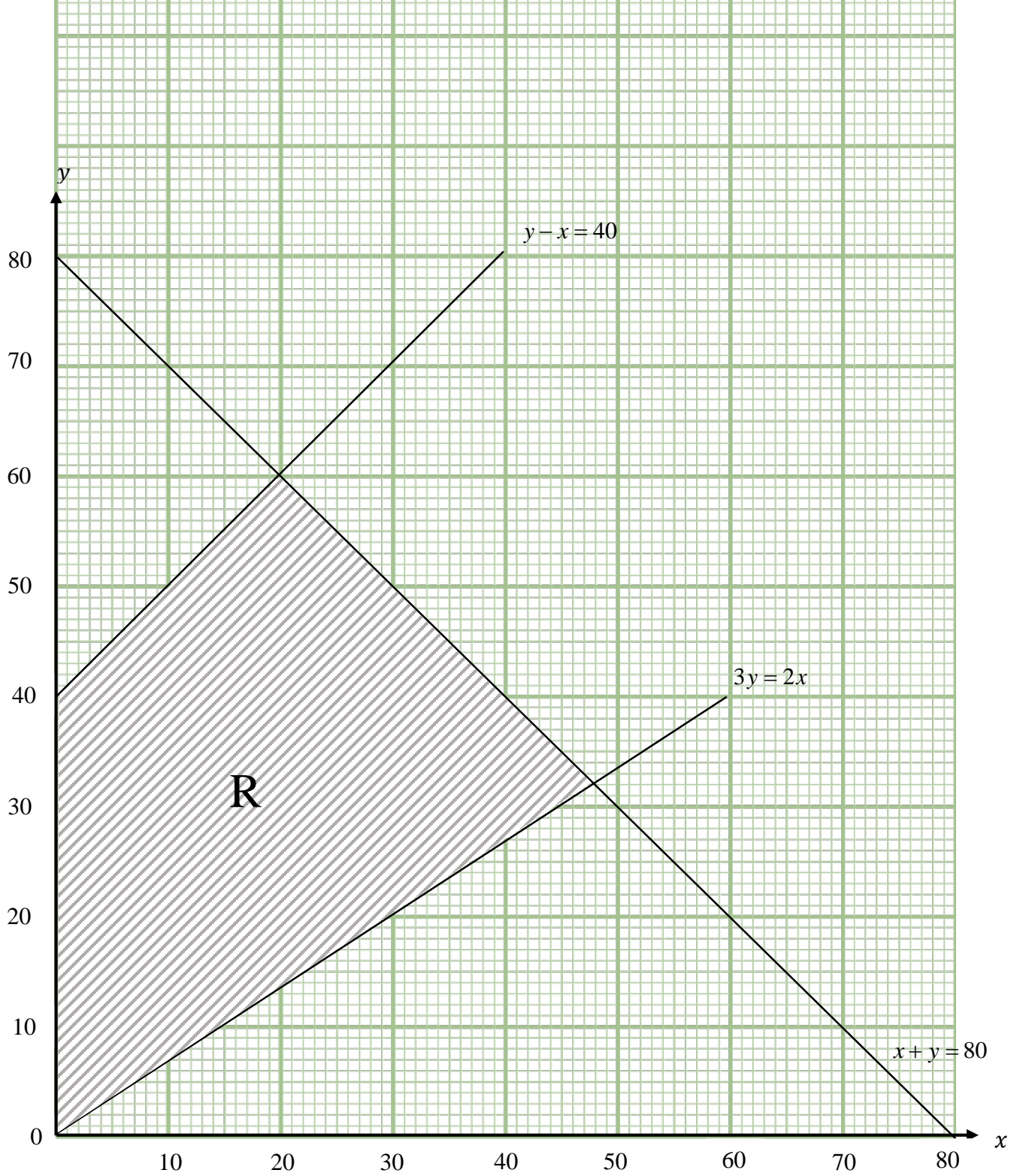


11. (a)	$x \times 2x \times h = 375$ OR $P = 4(x) + 4(2x) + 4h$	<div style="border: 1px solid black; display: inline-block; padding: 2px;">P1</div> Seen	3
	$P = 4(x) + 4(2x) + 4\left(\frac{375}{2x^2}\right)$	<div style="border: 1px solid black; border-radius: 50%; display: inline-block; padding: 2px;">K1</div> Substitute height in P	
	$P = 12x + \frac{750}{x^2}$	<div style="border: 1px solid black; display: inline-block; padding: 2px;">N1</div>	
(b) (i)	$12 - \frac{1500}{x^3}$	<div style="border: 1px solid black; border-radius: 50%; display: inline-block; padding: 2px;">K1</div> Differentiate P w.r.t. x	
	$12 - \frac{1500}{x^3} = 0$	<div style="border: 1px solid black; border-radius: 50%; display: inline-block; padding: 2px;">K1</div> Equate $\frac{dP}{dx} = 0$	
	$x = 5$	<div style="border: 1px solid black; display: inline-block; padding: 2px;">N1</div>	5
(ii)	$12(5) + \frac{750}{(5)^2}$	<div style="border: 1px solid black; border-radius: 50%; display: inline-block; padding: 2px;">K1</div> Substitute value of x in P	
	90	<div style="border: 1px solid black; display: inline-block; padding: 2px;">N1</div>	
(c)	$\frac{90}{100} \times 30 \times 1.80$	<div style="border: 1px solid black; border-radius: 50%; display: inline-block; padding: 2px;">K1</div>	
	$\text{RM}48.60$	<div style="border: 1px solid black; display: inline-block; padding: 2px;">N1</div>	2

<p>12 (a)</p>	<p>$a = 2ht + k$</p> <p>$0 = h(3)^2 + k(3)$ or $-3 = 2h(1) + k$</p> <p>$9 = 3h$ or $3k = -27$</p> <p>$h = 3, k = -9$ (both)</p>	<p>K1 Differentiate v w.r.t. t to find a.</p> <p>K1 Substitute $t = 3, v = 0$ or $t = 1, a = -3$ in the correct function</p> <p>K1 Solve simultaneous linear equation</p> <p>N1</p>																		
<p>(b)</p>	<p>$*3t^2 - *9t < 0$</p> <p>$3t(t - 3) < 0$</p>  <p>OR</p>  <p>OR</p> <table border="1" data-bbox="371 940 808 1073"> <thead> <tr> <th></th> <th>$t < 0$</th> <th>$0 < t < 3$</th> <th>$t > 3$</th> </tr> </thead> <tbody> <tr> <td>$3t$</td> <td>-</td> <td>+</td> <td>+</td> </tr> <tr> <td>$t - 3$</td> <td>-</td> <td>-</td> <td>+</td> </tr> <tr> <td>$3t(t - 3)$</td> <td>+</td> <td>-</td> <td>+</td> </tr> </tbody> </table> <p>$0 < t < 3$</p>		$t < 0$	$0 < t < 3$	$t > 3$	$3t$	-	+	+	$t - 3$	-	-	+	$3t(t - 3)$	+	-	+	<p>K1 Use $v < 0$</p> <p>K1</p> <p>N1</p>	<p>4</p> <p>3</p>	
	$t < 0$	$0 < t < 3$	$t > 3$																	
$3t$	-	+	+																	
$t - 3$	-	-	+																	
$3t(t - 3)$	+	-	+																	
<p>(c)</p>	<p>$\int *3t^2 - *9t dt = t^3 - \frac{9}{2}t^2 + c$</p> <p>$(2)^3 - \frac{9}{2}(2)^2$ or $(*3)^3 - \frac{9}{2}(*3)^2$ or $(5)^3 - \frac{9}{2}(5)^2$</p> <p>Use $s_3 - s_2 + s_3 + s_5$</p> <p>OR</p> <p>Use $\left \int_2^3 *3t^2 - *9t dt \right + \int_3^5 *3t^2 - *9t dt$</p> <p>$29.5 // \frac{59}{2}$</p>	<p>K1 Integrate and substitute t correctly</p> <p>K1 Find total distance</p> <p>N1</p>	<p>3</p>	<p>10</p>																

<p>13</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>$3y \geq 2x$ or equivalent</p> <p>$x + y \leq 80$ or equivalent</p> <p>$y - x \leq 40$ or equivalent</p> <p>Graph</p> <p>Draw correctly at least one straight line from *inequalities x and/or y.</p> <p>Draw correctly all the *straight line from *inequalities x and/or y.</p> <p>Region shaded correctly (refer graph)</p> <p>(i) 10</p> <p>(ii) (30, 50)</p> <p>$20(*30) + 30(*50)$</p> <p>2100</p> <p>Note: SS-1 once if</p> <p>(i) In (a) more than 3 inequalities given @ x and y not use at all</p> <p>(ii) In (b) does not use given scale @ axis interchange</p>	<p>N1</p> <p>N1</p> <p>N1</p> <p>K1</p> <p>N1</p> <p>N1 Note: Accept if region labeled R</p> <p>N1 No multiple answer</p> <p>N1</p> <p>K1 Substitute any point in the *shaded region into $50x+30y$</p> <p>N1 Note: point in integer only</p>	<p>3</p> <p>3</p> <p>4</p>	<p>10</p>
--	--	---	----------------------------	-----------

Graph for question 13



14
(a)

$$\sin \angle QRP = \frac{5.36}{6.99} \quad \text{OR} \quad \sqrt{6.99^2 - 5.36^2} \quad \text{OR}$$

$$\frac{1}{2} \times 12.01 \times 6.99 \times \sin \angle QRP = \frac{1}{2} \times 12.01 \times 5.36$$

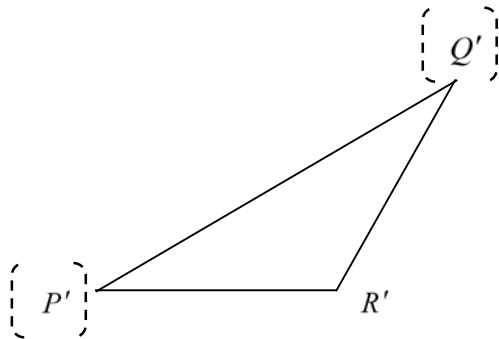
$$PQ^2 = (6.99)^2 + (12.01)^2 - 2(6.99)(12.01) \cos 50.07^\circ$$

OR

$$\sqrt{7.5233^2 + 5.36^2}$$

$$9.22 \leftrightarrow 9.24$$

(b) (i)



(ii)

$$129.93$$

(iii)

$$\frac{\sin \angle QPR}{6.99} = \frac{\sin \angle PRQ}{PQ} \quad \text{OR}$$

$$6.99^2 = (PQ)^2 + (12.01)^2 - 2(PQ)(12.01) \cos \angle QPR$$

$$\text{OR} \quad \angle QPR = \sin^{-1} \left(\frac{5.36}{9.2374} \right)$$

$$35.46 \leftrightarrow 35.55$$

K1

N1

K1

3

P1

Note:

- (i) $\angle P'R'Q'$ **must** be obtuse angle and sketch by using ruler.
- (ii) No multiple answer

N1

Note:

No multiple answer

K1

N1

4

(c)

$$\frac{1}{2}(12.01)\left(\frac{6.99}{2}\right)\sin * 50.07^\circ$$

$$\text{Area } \triangle ABR = \frac{2}{5}\left(\frac{1}{2}(12.01)\left(\frac{6.99}{2}\right)\sin * 50.07^\circ\right)$$

6.42 ↔ 6.44

Alternative method

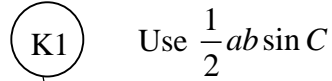
$$PB = \sqrt{(12.01^2) + \left(\frac{6.99^2}{2}\right) - 2(12.01)\left(\frac{6.99}{2}\right)\cos * 50.07^\circ}$$

$$PB = 10.1277$$

$$\left\{ \begin{array}{l} AB = \frac{2}{5}(10.1277) \end{array} \right.$$

$$\text{Area } \triangle ABR = \frac{1}{2} \left(\frac{2}{5}(10.1277) \right) \left(\frac{6.99}{2} \right) \sin 114.59^\circ$$

6.42 ↔ 6.44

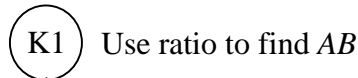


Use $\frac{1}{2}ab \sin C$

Use ratio to find area $\triangle ABR$



3



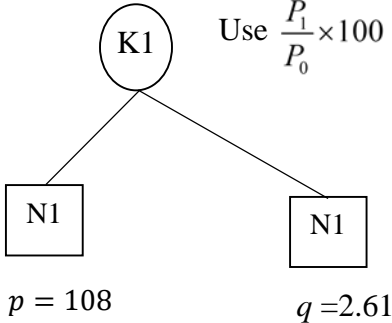
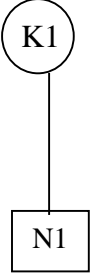
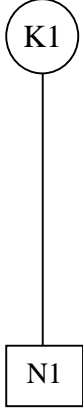
Use ratio to find AB

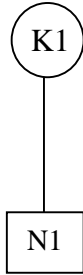
Use $\frac{1}{2}ab \sin C$



6.42 ↔ 6.44

10

15 (a)	$p = 108$ $q = 2.61$		3	
(b)	$I_{2\frac{1}{19}} = \frac{130 \times 112.5}{100}$ <p style="text-align: center;">146.25</p>		2	
(c)	<p>(i)</p> <p>54 @ 36 // 45 @ 30 @ 10</p> $\frac{*108(162) + 120(*54) + 145(108) + 130(*36)}{360}$ <p>or</p> $\frac{*108(*45) + 120(15) + 145(*30) + 130(*10)}{100}$ <p>123.10</p>	<p>P1 Seen or implied</p> 		

	<p>(ii)</p> $\frac{P_{20} + 2.50}{P_{20}} \times 100 = 110 \quad @ \quad \frac{P_{21}}{P_{21} - 2.50} \times 100 = 110$ <p>@ 2.50 x 10</p> <p>27.50</p>		5	10
--	---	---	---	----